



DARK MATTER IN THE CLASSROOM: A SPARC-BASED ROTATION CURVE EXERCISE FOR TEACHERS

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We present an educational module in which students use real SPARC data (Spitzer Photometry & Accurate Rotation Curves) in Python and MS Excel or LibreOffice Calc. They build rotation curves for disk galaxies, compare observed circular velocities with the baryonic contribution, and use a simple Newtonian mass estimate to infer the need for additional unseen matter. The activity connects programming, data analysis and introductory theoretical physics through a hands-on classroom exercise. The idea for this exercise was inspired by the Winter School of Physics, organized by the Faculty of Sciences and Mathematics, University of Niš, where students engaged with physics through hands-on activities.

BASIC IDEAS:

Goal: Identify and quantify the need for dark matter with Python and Excel/Calc in a classroom exercise.

Data retrieval: Students use real data from the SPARC database [1,2]. The data can be downloaded manually in ZIP format from <https://astroweb.case.edu/SPARC/> or using Python script to retrieve the data, available at Google Colab (**the QR code for Google Colab with script is located in the upper-right corner of the poster**).

Data processing: After downloading data, students import selected galaxy tables into Excel/Calc and calculate derived quantities.

Analysis: The final graphs compare V_{obs} and V_{bar} , estimate the dynamical mass, and show the inferred dark-matter fraction.

This simplified approach is useful for teaching, however, it has several limitations:

Circular motion: Measured gas velocities are interpreted as approximately circular rotation; bars, warps and non-circular motions can introduce deviations.

Spherical approximation: The equation $M \approx RV^2/G$ is a simple mass estimate. Real galaxies contain flattened stellar and gas disks.

Galaxy inclination: Line-of-sight velocities must be corrected for the orientation of the galaxy, which introduces uncertainty.

Small radii: Limited spatial resolution and complex motions can make the inner rotation curve less certain.

Mass-to-light ratio (M/L): The stellar mass estimate depends on the adopted M/L ratio, which is uncertain.

EXCEL TABLE

The processed data are arranged in columns for

$$R, M_{obs}, M_{bar}, v_{obs} \text{ and } v_{gas}.$$

By selecting the required columns, we create the necessary graphs. On the first graph, we manually add the Keplerian curve by entering the radius values and calculating the corresponding velocities using the v_{flat} from CSV.

CONCLUSION

This exercise supports active learning by combining physical concepts, real observations, and simple computational tools.

Its main educational value is that students not only learn about dark matter as a concept, but reconstruct the argument from real observational data.

In the outer region, if the enclosed mass is approximately constant, the circular velocity decreases as the inverse square root of distance:

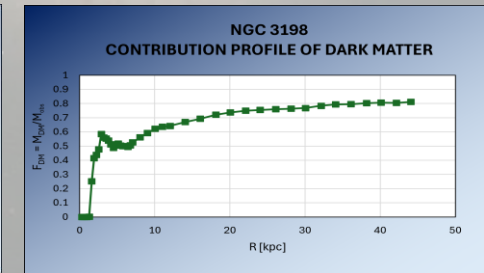
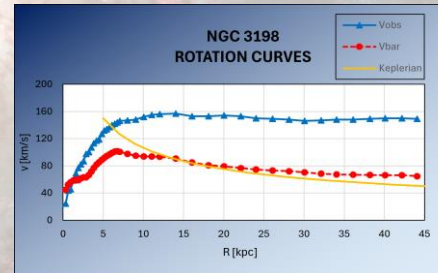
$$v(R) = \sqrt{\frac{GM_{bar}}{R}} \quad R > R_{disk} \rightarrow M_{bar} = const.$$
$$\frac{dv(R)}{dR} = -\frac{1}{2} \sqrt{GM_{bar}} \frac{1}{R^{\frac{3}{2}}} \quad \frac{dv(R)}{dR} < 0$$

PHYSICS FOUNDATIONS

According to Newtonian gravity, the circular velocity depends on the gravitating mass and the distance from the galactic center. If most gravitating mass were concentrated within the visible galaxy, the enclosed mass would become approximately constant at large radii, and the velocity would decrease as $v \propto R^{-1/2}$: a Keplerian decline. **Observed rotation curves instead remain nearly flat far beyond the visible disk.** In the standard Newtonian interpretation, this indicates additional unseen matter - dark matter. In this exercise, we calculate the baryonic contribution from gas and stars, estimate the dynamical mass using the spherical approximation $M_{obs} \approx Rv_{obs}^2/G$, and infer an excess mass $M_{DM} \approx M_{obs} - M_{bar}$ [3]. The plot on the right shows the ratio of dark matter to observable matter, i.e. the dark matter fraction $f_{DM} = M_{DM}/M_{obs}$.

WORKING EXAMPLE

While this method is applicable to the entire SPARC sample, the Python script has been implemented for a select subset of galaxies. As an example, we show the results for NGC 3198.



MAIN EQUATIONS:

From the equality of the gravitational and centripetal forces, we obtain the relation for the mass enclosed within a given radius. This equation can be rewritten using more convenient units, with velocity in km/s instead of m/s, radius in kpc instead of meters and kilograms in $M_{\odot}$ by introducing the appropriate conversion factors:

$$G \frac{M_{obs}(R)}{R^2} = \frac{v^2(R)}{R} \quad M_{obs}(R) = \frac{Rv^2(R)}{G} \quad M_{obs}[10^9 M_{\odot}] = \frac{v^2[\frac{km}{s}]R[kpc]}{4302}$$

The baryonic contribution to the rotation curve includes gas, the stellar disk, and, where present, a central bulge. The stellar contributions are scaled by the adopted mass-to-light ratios, Y_d and Y_b . The total baryonic rotation velocity is therefore estimated as:

$$v_{bar}(R) = \sqrt{v_{gas}^2 + Y_d v_{disk}^2 + Y_b v_{bul}^2}$$

[1] SPARC database: <https://astroweb.cwru.edu/SPARC/>

[2] Lelli, F., McGaugh, S. S., Schombert, J. M., *SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves*, AJ 152, 157 (2016)

[3] Binney, J., Tremaine, S., *Galactic Dynamics*, 2nd ed., Princeton University Press (2008)